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Local Buckling and Mode Switching in the Optimum Design of Stiffened Panels

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Local buckling can significantly affect the strength of stiffened panels. Most studies on the optimization of stiffened panels consider only one or two forms of buckling (buckling modes), because considering all possible mode shapes in nonlinear analysis of stiffened panels, particularly during the iterative process of optimization, is a complex and time-consuming task. This study presents an approach for optimization of stiffened panels considering geometric nonlinearity and local buckling. A method is presented to efficiently incorporate the effects of local buckling and mode switching during the optimization process.

I. Introduction

STRUCTURAL optimization aims to find an optimum shape or topology for the structure to minimize or maximize an objective function while satisfying a set of design constraints. Structural optimization to increase the buckling resistance has always been an active area of research. Most of the works in this area deal with linear buckling analysis; however, the deformations of many thin-walled structures under applied loads are beyond the range of linear deformations. As a result, the exact strength of the structure can only be found using nonlinear analysis.

Among the first publications on the nonlinear large-deflection postbuckling behavior of unstiffened rectangular plates are the publications by Marguerre [1], Kromm and Marguerre [2], and Levy [3]. Paik et al. [4] captured the nonlinear large-deflection response of unstiffened plates by an incremental Galerkin method, an approach previously outlined by Ueda et al. [5]. Paik et al. [6,7] also considered stiffened plate with stiffeners "smeared" on the main plate. In this case, the stiffened plate is modeled as an orthotropic unstiffened plate. Byklum et al. [8–10] studied stringer-stiffened plates with large deflections. In those studies, stringers are not smeared on the plate, but are considered to be structural elements.

The problem of interaction of an Euler buckling with local plate buckling was studied by several researchers [11–15]. Tvergaard [16] presented a detailed analysis of stiffened plates under interactive buckling. Byskov and Hutchinson [17] used an asymptotic approach for the same problem.

Comparisons between studies on the ultimate strength for stiffened panels have been presented in [18–21]. Experimental investigation for the collapse behavior of stiffened panels can be found in [22–24]. An extensive contribution to the ultimate-strength design for ships' stiffened panels has been provided in [25–28]. The effect of combined axial compression and lateral loads has been studied by Hughes and Ma [29,30] and Hu et al. [31].

Research works on shape optimization can be found in [32–42]. There are also studies on the sensitivities of limit points of nonlinear structures [42–49]. Some works dealing with bifurcation points [50,51] do not specifically address finite-element-method applications, whereas others are restricted to the semi-analytical approach [52,53]. Also, closed-form solutions, simple physical models, and finite strip methods have been extensively used for design optimization of stiffened panels [54–59].

Considering the appropriate imperfection in nonlinear analysis is a key part of the optimization process. The form of the imperfection affects the nonlinear response, and as a result, the final optimum shape also depends on the assumed imperfection. The answer to the question of which form of imperfection is the best to consider is still unknown. The most popular approach is to perform the nonlinear analysis by considering an imperfection similar to the first buckling-mode shape. However, there is no guarantee that the first buckling-mode shape remains unique throughout the optimization process. This means that the first buckling mode of the optimum design may be completely different from the assumed imperfection.

The objective of this study is to present an approach for optimization of stiffened panels considering geometric nonlinearity and local buckling. A method is presented to efficiently perform the optimization task and to incorporate the effect of possible buckling modes as the initial imperfection in the nonlinear analysis. Illustrative examples are presented.

II. Buckling Modes of Stiffened Panels

Figure 1 shows the buckling modes of a typical stiffened panel. According to [60], the primary failure modes for a stiffened panel subject to compressive loads are categorized into the following six modes:

1) Mode a is the overall (global) buckling of the plate and stiffeners. This mode typically happens when the stiffeners are relatively weak, and as a result, they buckle together with the plate.

2) Mode b is the local buckling of the plate between the stiffeners. In this mode, the panel collapses due to the local buckling and consequent yielding of the plate between the stiffeners.

3) Mode c is the beam–column-type buckling of the combination of stiffener and effective plate. In this mode, the failure happens by beam–column-type collapse of the combination of stiffener and the associated effective (reduced) plate.

4) Mode d‡ is the local buckling of the stiffener web. This mode is usually called the "stiffener-induced" failure mode.

5) Mode e‡ is the lateral–torsional buckling of the stiffener web. This mode is similar to mode d, except that buckling of the stiffener is a lateral–torsional (tripping) buckling.

6) Mode f is the entire yielding of the panel cross section. This mode usually happens when the panel slenderness is very small or when the panel is subjected to the tensile load. In this case, the panel cross section yields entirely, without any local or overall buckling.

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‡Modes d and e typically happen when the height-to-thickness ratio of the stiffener web is too large and/or when the stiffener flange (e.g., in Z stiffeners) is inadequate to keep it straight.

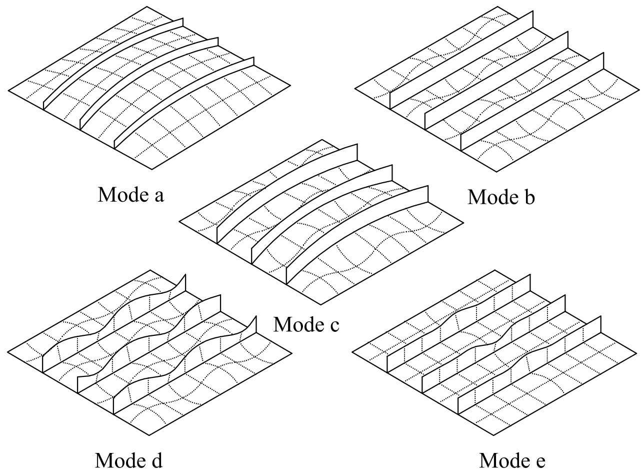

Fig. 1 Primary buckling modes of a stiffened panel subject to compressive loads [60].

For a stiffened panel subject to compressive loads, lateral deformations cannot be initiated unless an imperfection is considered in the analysis. During the optimization process, the shape of the stiffened panel is changed gradually, and each time that the shape is modified, another analysis is required. Most designers consider an imperfection similar to one of the failure modes during the whole process. This is obviously not a reliable procedure, and we try to modify that by updating the failure mode during optimization. Before going to the optimization methodology, a review of the bifurcation buckling and nonlinear analysis is presented in the next section.

III. Bifurcation Buckling and Nonlinear Analysis

Figure 2 shows the behavior of the perfect and imperfect stiffened panels under axial loads. In a perfect structure, the equilibrium path reaches a bifurcation point and follows the second branch as the nonlinear postbuckling equilibrium path. In case the structure is not perfect, the equilibrium path falls bellow that curve, with a margin depending on the amount of imperfection. Because the deformations of a perfect structure before the bifurcation point are small, load P_B may be found with a high accuracy using a linear buckling analysis. However, this is not true for an imperfect structure, because the limit load P_L may only be found by performing a nonlinear analysis.

Two forms of bifurcation points are usually observed in equilibrium paths of stiffened panels under axial loads. The first type (depicted in Figs. 2a and 2b) is a form of symmetric bifurcation, in which the behavior or strength of the imperfect structure (i.e., load P_L) does not depend on the direction of imperfection. In this case, (which usually happens in buckling modes b, d, and e), nonlinear analysis of the structure with a small imperfection in the form of the buckling mode with either positive or negative sign leads to the same result as the limit load of the imperfect structure. The second form of the bifurcation point is usually observed in global buckling (and in mode c), in which the direction of the global bending makes a difference in the behavior (and, consequently, the strength) of the structure, as shown in Fig. 2c. In this case, two analyses are usually required for the imperfect structure, with small imperfections in the positive and negative directions.

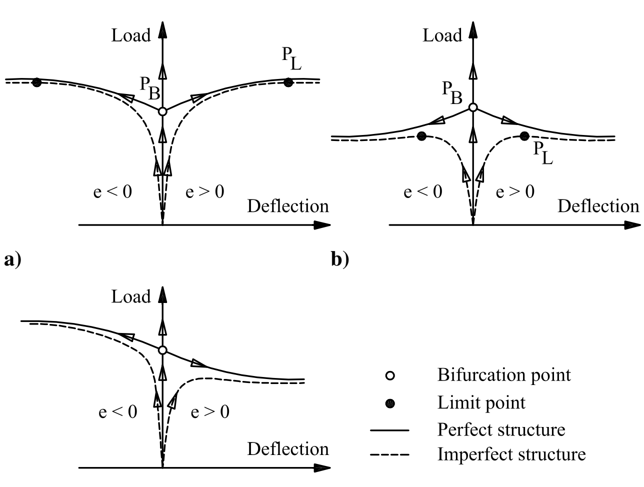

Fig. 2 Behavior of the perfect and imperfect stiffened panels.

A variety of methods and programs are available for the analysis of stiffened panels, ranging from simple closed-form solutions to complicated 3-D discretized solutions. The more complicated or detailed modeling usually employs discretized models such as finite element and boundary element analysis. To simplify the process, instead of the whole structure, a single module of the stiffened panel with appropriate boundary conditions is usually considered based on the repeating stiffener pattern (Fig. 3). This model has already been used (e.g., in [61]) and has been proven to be accurate enough.

As mentioned before, in the case of perfect structure, bifurcation buckling load P_B and its corresponding mode shape may be found by performing a simple linear eigenvalue analysis:

$$([K] + \lambda_i[K_\sigma])\phi_i = 0 \tag{1}$$

where $[K]$ and $[K_\sigma]$ are the material stiffness and stress stiffness matrixes in the global coordinate system, respectively, and ϕ_i represents the ith buckling-mode shape. The smallest λ_i and the corresponding ϕ_i are the load multiplier for the first bifurcation buckling load P_B and the first buckling-mode shape, respectively.

In the case of nonlinear analysis of an imperfect structure, the internal and external forces must be in equilibrium. Equilibrium equations of a finite element model can be written as

$$r(u, \lambda) = g(u) - \lambda f = 0 \tag{2}$$

where r, g, and f are the vectors of the unbalanced load, internal force, and reference load, respectively; u is the vector of the nodal displacements, and λ is the load factor. The simplest procedure to trace the equilibrium path is the load-control method combined with Newton–Raphson iteration. However, the load-control method can trace the load-displacement curve before the occurrence of a limit point (i.e., P_L). To capture the limit load accurately, one should use other path-following methods such as the displacement-control method [62] or various forms of the arc-length method [63,64].

In this study, an efficient facet triangular shell element with three nodes and six degrees of freedom per node is used to model the structure. This element is a combination of the discrete Kirchhoff triangular plate-bending element [65] and the optimal triangular membrane element [66]. It has been shown in a recent paper by the authors [67] that by using this element, one can get acceptable results even in the case of relatively coarse meshes. This means that the cost of the optimization of nonlinear structures may be extensively reduced using this element. The reason is that the response of this

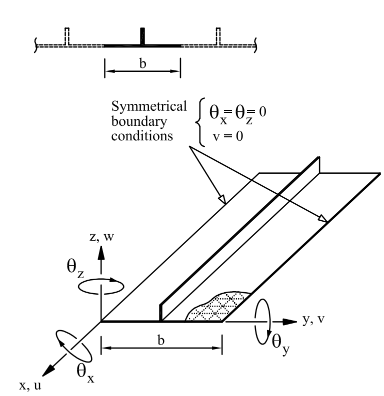

Fig. 3 Discretized panel module used in finite element analysis.

element is less dependent on the size of the mesh or on the element geometrical aspect ratio (particularly, in the case of in-plane deformations). This is an important issue in analysis of stiffened panels, because in-plane deformations usually happen in the web, and using other elements to model them may lead to significant computational errors.

Nonlinear finite element analysis is performed using the corotational approach combined with the arc-length method. In the corotational approach, the contribution of the rigid-body motion to the total deformation of the element is removed before performing the element computations. For a detailed explanation of corotational nonlinear analysis using facet triangular elements, the reader is referred to [67]. Analysis is performed until the stress level reaches a maximum allowable value or the path reaches a limit point in the form of a snap-through. In the case of a snap-through, analysis is stopped when the load level at a newly converged point of the path becomes less than that of the previous converged point (which means that the limit point has been passed). Then, using the values corresponding to the last three points, a quadratic curve is fit to the load-displacement curve near the limit point, and the limit load is found with high accuracy [39].

IV. Optimization Methodology

In this section, the shape optimization problem is described in more detail. Considering X as the vector of the shape design variables, the optimization problem is to maximize the limit load P_L subject to the constant volume:

$$h(X) = V - V_0 = 0 \qquad X_L \leq X \leq X_U \tag{3}$$

where $h(X)$ is the equality constraint, X_L and X_U are the lower and upper bounds on the design variables X, and V and V_0 are the total volume and the constant value for the total volume. The Lagrangian can be defined as

$$L(X, \mu) = -P_L + \mu h \tag{4}$$

where μ is the Lagrange multiplier. The stationarity conditions of the Lagrangian function (Karush-Kuhn-Tucker conditions [68]) together with the equilibrium equation (2), describe the optimization problem:

$$-\nabla_X P_L + \mu \nabla_X h = 0 \qquad h(X) = 0 \qquad r(u, \lambda) = 0 \tag{5}$$

where ∇_X is the total derivative. The solution of this nonlinear system of equations defines the optimum design. The optimization process usually starts with the nonlinear analysis of an initial design. At the limit load, the sensitivity analysis is performed and a search direction ΔX is produced, which is used in the line-search procedure. Using the value $\alpha = 1.0$ for the step length and letting $X = X + \alpha \Delta X$, a new nonlinear analysis is performed. If there is no improvement, the line-search procedure generates a new step length α and the structural analysis is repeated until an improvement is obtained.

It can be seen that this solution consists of two main parts, as shown in Fig. 4a. The first part is the nonlinear finite element analysis (NFEA), and the second part is the optimization in which the design variables are modified. These two parts are repeated until convergence is achieved. One can see that this is a very time-consuming task, because the iterative process of NFEA has to be performed inside another iterative process (optimization loop).

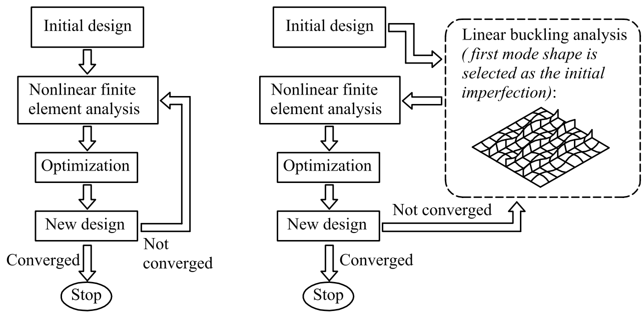

Fig. 4 Algorithms for optimization of nonlinear structures.

As mentioned before, for nonlinear analysis of imperfect structures, an imperfection in the form of a buckling mode is usually considered. Obviously, performing NFEA for all buckling modes shown in Fig. 1 (in every iteration of optimization) makes the optimization process more complex and practically impossible to perform. On the other hand, selecting only one buckling mode as the imperfection (which is usually the first buckling mode of the initial design) and using that imperfection during the whole optimization process is not justified, due to the constant change of the shape. As a result, a criterion should be employed to efficiently modify the imperfection as the shape of the structure is changing.

Figure 4b shows how the optimization algorithm can be modified to consider the change of buckling mode and corresponding imperfection. Each time before performing the NFEA, a linear eigenvalue analysis is performed and the smallest linear buckling load (load P_B in Figs. 2a and 2b) and its corresponding mode shape for the perfect structure are found. Then a small imperfection, similar to that in the buckling-mode shape, is considered in the NFEA. Performing the NFEA, the critical load of the structure is then considered as $P_{cr} = P_L$, where P_L is the limit load of the imperfect structure. Using this methodology, NFEA is performed only for the first buckling mode in each iteration of the optimization process, and mode switching due to the change of the shape (during the optimization process) can also be captured. It means that if the first buckling-mode shape changes during the optimization, the imperfection also changes, and the imperfection assumed in the beginning of the solution is not kept throughout the whole process.

It should be mentioned that in the case of stress constraint, the load capacity of the structure should be reduced to the load corresponding to the allowable stress, if the stress constraint becomes active prior the limit load. Regarding the sensitivity analysis, the reader is referred to an analytical method in [49] that is independent of the element type and is valid for shape and size design variables. The finite difference method (FDM) may also be used as an alternative approach for sensitivity analysis:

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{6}$$

The accuracy of the FDM depends on the perturbation size Δx. A fixed value cannot be used for Δx because it is problem-specific; however, a relative perturbation between $10^{-5}x$ and $10^{-8}x$ usually leads to the results with sufficient accuracy.

V. Numerical Examples

In this section, two numerical examples are presented to demonstrate the described method. The first example is basically a one-variable problem and allows us to focus on the aspects of local buckling during the change of the shape without going through the details of the optimization process. The second example is a multivariable problem and is solved using the gradient-based sequential quadratic programming [68] method of optimization.

A. Example 1: One-Variable Design of a Stiffened Panel

As the first example, the stiffened panel shown in Fig. 5 is considered. This example, which has already been investigated in the literature [16,53], is studied here under both stress and stability constraints. The panel is infinitely wide, and only a single module of the panel with the boundary conditions shown in Fig. 4 is considered with the dimensions and material properties given in Fig. 5. Thickness of the plate and stiffeners (t_p and t_s) are considered as the design variables. The objective is to maximize the load capacity of a single module of this stiffened panel subject to the volume equality constraint $V = 65,820\ \mathrm{cm}^3$, and stress constraint $\sigma_{VM} \leq 25,000\ \mathrm{N/cm}^2$ (with σ_{VM} representing the von Mises stress). The inequality side constraint $12\ \mathrm{mm} \leq t_p \leq 18\ \mathrm{mm}$ is also considered

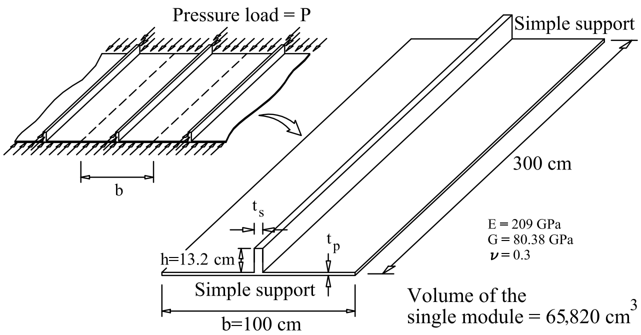

Fig. 5 Stiffened panel under axial load.

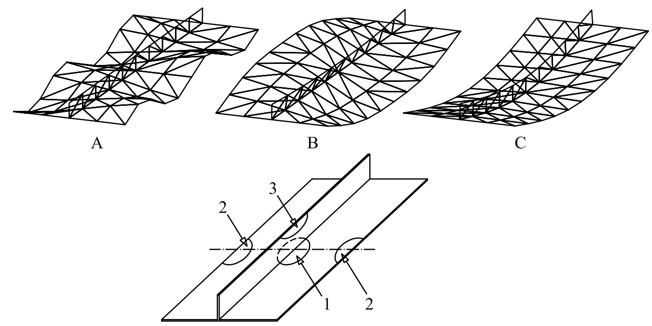

Fig. 6 Buckling modes and yielded regions for the stiffened panel in example 1.

for the plate thickness. This problem may actually be reduced to a one-variable problem, because the thickness of the stiffener may be obtained when changing the plate thickness, using the volume equality constraint. Thus, it is possible to analyze the panel within the range 12 mm $\leq t_p \leq$ 18 mm for several thicknesses of the plate and corresponding stiffener thicknesses.

Table 1 shows the result of analysis for different values of t_p listed in the first column. For each case, the first bifurcation load P_B and the corresponding mode shape are found by the linear buckling analysis of the single module, as shown in the second column of the table (mode shapes A, B, and C are shown in Fig. 6). Then, for each case, nonlinear analysis is performed considering an imperfection similar to the first buckling mode with the maximum value of 1 cm. This value is assumed to be the smallest amount of imperfection that may be detected with the naked eye during the manufacturing of this panel. Load capacity of the imperfect stiffened panel is found based on the stress constraint and stability criterion and is shown in the third column of the table (P_{cr}). It is seen that in this example, a region in the imperfect structure always reaches the maximum stress before reaching the nonlinear limit load. The region in which yielding happens first in the imperfect structure is noted in parentheses in the third column of the table and is also shown in Fig. 6. Load capacities P_{cr} for different designs are plotted in Fig. 7. It is observed that the design with $t_p = 16$ (mm) has the highest load capacity and is therefore the optimum design.

As seen for the lower values of t_p, the stiffened panel fails by local buckling of the panel; however, as t_p is increased, the first buckling-mode shape changes, and at $t_p = 18$ mm, the structure tends to fail by global buckling. This example shows that as the shape of the structure changes during the optimization process, failure mode may also change as well, and a single unique failure mode may not be considered as the imperfection throughout the whole process.

Table 1 Results of the analysis of the stiffened panel for different values of t_p

t_p, mm	P_B, N/cm^2 [a]	P_{cr}, N/cm [b]
12.0	13,918 (A)	12,618 (1)
12.5	15,076 (A)	12,862 (1)
13.0	16,265 (A)	13,170 (1)
13.5	17,475 (A)	13,408 (1)
14.0	18,694 (A)	13,644 (1)
14.5	19,905 (A)	13,800 (1)
15.0	21,085 (A)	13,965 (1)
15.5	22,201 (A)	14,058 (1)
16.0	22,164 (B)	23,148 (2)[c]
16.5	21,356 (B)	21,489 (2)
17.0	20,733 (B)	22,209 (2)
17.5	20,359 (B)	21,567 (2)
18.0	19,600 (C)	9489 (3)

[a]The letters in parentheses denote the buckling-mode shape, as shown in Fig. 6.
[b]The numbers in parentheses denote the yielded region, as shown in Fig. 6.
[c]Optimum design.

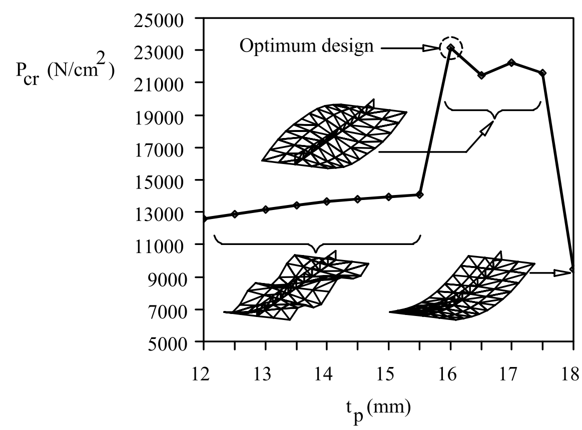

Fig. 7 Load capacity (different designs) for stiffened panel in example 1.

This example problem has already been considered without stress constraint in [53], leading to the value of $t_p = 13$ mm and $P_L = 22650$ N/cm^2 as the limit load of the structure based on an imperfection in the form of the global buckling (mode C in Fig. 6). However, as it may be seen in Fig. 7, the first buckling mode for $t_p = 13$ mm is a local buckling (mode A in Fig. 6). This means that as the load is applied on the panel, local buckling happens before the global buckling. One can conclude that in this case, nonlinear analysis based on an imperfection in the form of global buckling is not justified, and the result found based on that assumption is unreliable. Also it may be seen that it is not feasible to ignore the stress constraint in design optimization of stiffened panels, because as is clear from Table 1 for thickness $t_p = 13$ mm, local yielding will occur in region 1 at a load level much lower than the limit load.

B. Example 2: Multivariable Design of a Stiffened Panel

A stiffened panel similar to that in the previous example with a constant length of 250 cm and four design variables b, t_p, h, and t_s is considered. The objective is to maximize the limit load of this stiffened panel subject to the volume equality constraint. Although it was shown in the previous example that it is not practical to ignore the stress constraint in the optimization of stiffened panels, no stress constraint is considered in this example, to show the effect of mode switching on the limit-load maximization. Here again, the first bifurcation load and the corresponding mode shape are found by the linear buckling analysis of the single module, and then nonlinear analysis is performed considering a small imperfection similar to the first buckling mode. Optimization is performed using the gradient-based sequential quadratic programming method of optimization, and sensitivity analysis is performed by the finite difference method. Figure 8 shows the iteration history, along with three sample points (A, B, and C). Optimization process starts with $b = 80$ cm, $t_p = 1$ cm, $h = 10$ cm, and $t_s = 1$ cm as the initial point and leads to

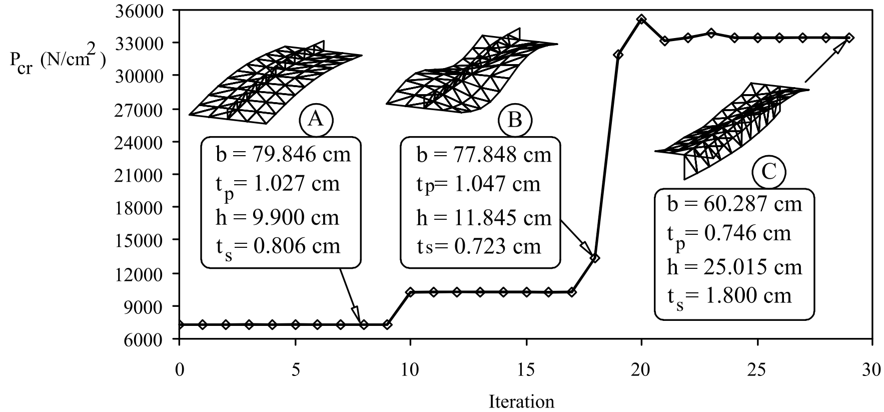

Fig. 8 Iteration history and the change of the lowest buckling-mode shape for the stiffened panel in example 2.

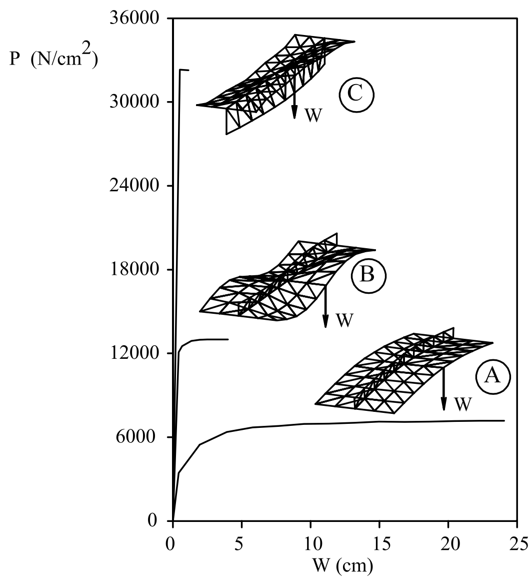

Fig. 9 Change of the nonlinear behavior during optimization of the stiffened panel in example 2.

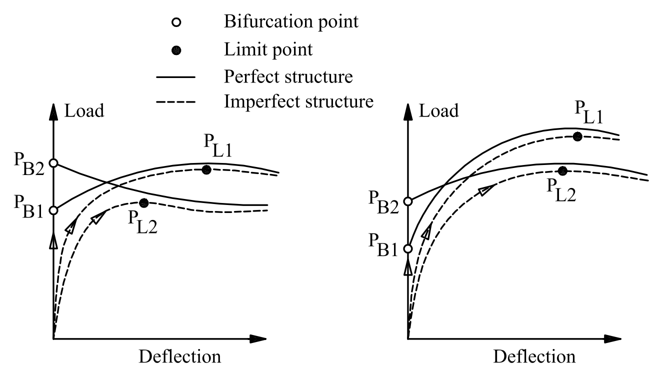

Fig. 10 Two other cases of bifurcation points and corresponding limit loads.

the optimum design with $b = 60.287$ cm, $t_p = 0.746$ cm, $h = 25.015$ cm, and $t_s = 1.800$ cm (point C). Figure 9 shows the nonlinear behavior of three sample points (A, B, and C). Again, it is seen that a unique failure mode may not be considered throughout the whole optimization process, and the proposed method captures the mode switching that happens during the optimization process due to the change in shape of the structure.

VI. Conclusions

The effect of local buckling in the optimization of stiffened panels is studied and a robust method of optimization is presented. In this method, the change of the first buckling-mode shape during the optimization process is considered, and the form of imperfection in the nonlinear analysis is modified as the first buckling mode (whether it is local or global) changes. It is shown that optimization of nonlinear stiffened panels may be performed efficiently using the proposed method.

It should be mentioned again that the lowest bifurcation point does not always lead to the lowest limit load, as seen in Fig. 10. In such cases, nonlinear analysis using an imperfection in the form of another buckling mode (rather than the first mode) leads to a lower limit load and, as a result, underestimates the strength of the structure. The present study is valid when no information is provided regarding the imperfection of the structure and the designer needs to consider some imperfection to investigate the nonlinear behavior. In this case, it is assumed that the structure follows the nonlinear equilibrium path that derives from the lowest bifurcation point.

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